Recent Developments in PDE Solvers using Overture

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13th Symposium on Overset Composite Grids and Solution Technology,
Future of Flight Aviation Center, Mukilteo, Washington, October 17–20, 2016.

Acknowledgments.

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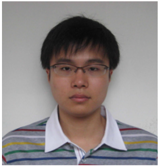

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Supported by:
National Science Foundation, RPI, LLNL.
Department of Energy, Office of Science, ASCR Applied Math Program.

Outline

1. fluid-structure interactions
2. conjugate heat transfer
3. upwind schemes for Maxwell's equations

Overture and the CG solvers source code

The Overture framework and Composite Grid (CG) PDE solvers are open source and available from (documentation, downloads)

`overtureFramework.org`

The source-code repositories are hosted at

`sourceforge.net/projects/overtureframework`

Monolithic vs Partitioned Schemes

Monolithic FSI Solver

Fluid + solid PDEs

Interface conditions:

$$\boxed{\begin{aligned} D &: \; [v] = 0 \\ N &: \; [\mathbf{n}^T \sigma] = 0 \end{aligned}}$$

Partitioned FSI Solver

Solid Solver

Solid PDE

Solid BC : ?

Fluid Solver

Fluid PDE

Fluid BC : ?

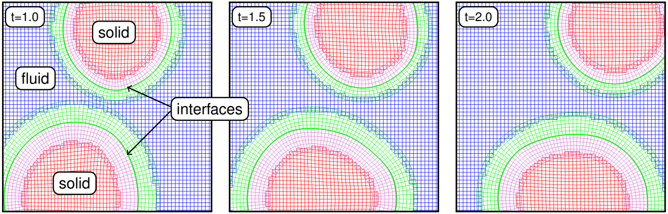
Objective: Choose the solid BC and fluid BC so the partitioned scheme is stable and accurate.

Fluid Structure Interaction (FSI) algorithms

Our approach in Overture is based on deforming composite grids.

Approach:

- Fluids are solved in an Eulerian frame.
- Solids are solved in a Lagrangian frame.
- Deforming interface grids are regenerated at each time-step with the hyperbolic grid generator.



Basic Components used in FSI Time-Stepping.

1. HyperbolicMapping - hyperbolic component grid generator (to recompute grids next to deforming surfaces).
2. Ogen grid generator - overset grid generator (compute connectivity).
3. Fluid solver (e.g. Cgcns, Cgins).
4. Solid Solver (e.g. Cgsm, BeamSolver, . . .)
5. Multi-physics, multi-domain driver program (Cgmp)

Basic FSI Algorithm.

procedure $\textsc{solveFsiDcg}(\mathcal{G}, t_{\text{final}})$ ▷ Input: initial composite grid and final time

$t := 0;\ n := 0;\ \mathcal{G}^n = \mathcal{G};$

$\texttt{assignInitialConditions}(\mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}^n);$

while $t < t_{\text{final}}$ **do**

$\Delta t := \texttt{computeTimeStep}(\mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}^n);$

$\mathcal{G}^p := \texttt{moveGrids}(\mathcal{G}^n, \mathbf{q}_\mathbf{i}^n, \bar{\mathbf{q}}_\mathbf{i}^n);$ ▷ (calls HyperbolicMapping)

$\mathcal{G}^p := \texttt{updateOverlappingGrid}(\mathcal{G}^p);$ ▷ (Ogen)

$\mathbf{q}_\mathbf{i}^{n+1} := \texttt{advanceFluid}(\mathbf{q}_\mathbf{i}^n, \mathcal{G}^n, \mathcal{G}^p, \Delta t);$

$\bar{\mathbf{q}}_\mathbf{i}^{n+1} := \texttt{advanceSolid}(\bar{\mathbf{q}}_\mathbf{i}^n, \mathcal{G}, \Delta t);$

$(\mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I) := \texttt{projectInterface}(\mathbf{q}_\mathbf{i}^{n+1}, \bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}^p);$

$\mathbf{q}_\mathbf{i}^{n+1} := \texttt{applyFluidBCs}(\mathbf{q}_\mathbf{i}^{n+1}, \mathcal{G}^p, \mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I);$

$\bar{\mathbf{q}}_\mathbf{i}^{n+1} := \texttt{applySolidBCs}(\bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}, \mathbf{n}^T\mathbf{v}^I, \mathbf{n}^T\boldsymbol{\sigma}^I);$

$\mathcal{G}^{n+1} := \texttt{correctMovingGrids}(\mathbf{q}_\mathbf{i}^{n+1}, \bar{\mathbf{q}}_\mathbf{i}^{n+1}, \mathcal{G}^p, \Delta t);$

$t := t + \Delta t;\quad n := n + 1;$

end while

end procedure

Deforming grid example

Incompressible flow past a beam

beam grid

/Users/henshaw/movies/beamInChannelGrid.mp4

Deforming grid example

Compressible flow past two elastic disks

multi-disk grid

/Users/henshaw/movies/shockMultiDiskGrid.mpg

multi-disk grid

/Users/henshaw/movies/shoc

Overture commands for a deforming grid simulation

```
PlotStuff ps(plotOption,"deform"); // for plotting

CompositeGrid cgNew, cgOld; // overset grids

Ogen ogen(ps);          // overset grid generator

HyperbolicMapping hyper; // Hyperbolic grid generator

// Supply a new surface grid:
NurbsMapping surface;
hyper.setSurface(surface,isSurfaceGrid,init);

// generate hyperbolic volume grid
hyper.generate();

// Generate overset grid connectivity
ogen.updateOverlap(cgNew, cgOld, hasMoved, option );

realCompositeGridFunction u(cgNew); // grid function

PlotIt::contour(ps,u); // contour and surface plots
```

Compressible flow + beam

CNS+beam

/Users/henshaw/movies/beamAndCompressibleF

Compressible flow + nonlinear elastic solid

Impedance weighted interface conditions provide stability for light solids

ellipse

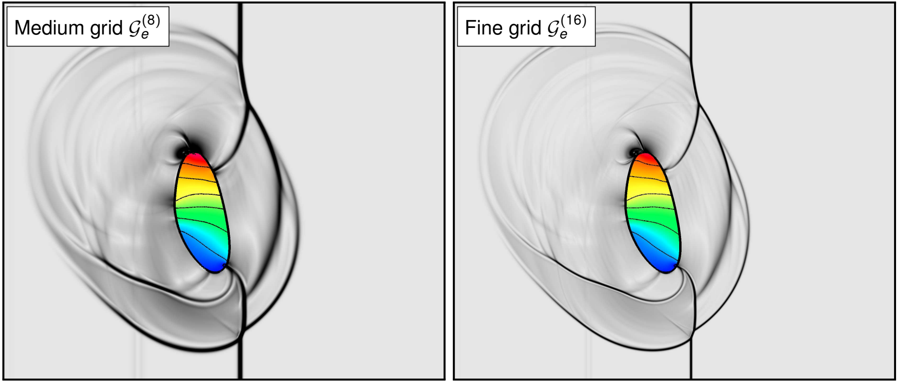
solid beam

/Users/henshaw/movies/defor

- J.W. Banks, WDH, A.K. Kapila, D.W. Schwendeman, *An Added-Mass Partitioned Algorithm for Fluid-Structure Interactions of Compressible Fluids and Nonlinear Solids*, J. Comput. Phys. (2016).

Shock impacting an elliptical solid

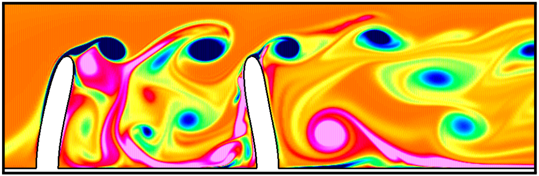
Grid convergence



AMP scheme for incompressible flow + beam-model

Fluid structure interactions

1. A pressure-velocity formulation of the incompressible flow equations are solved with a second-order fractional-step scheme.
2. A special Robin (mixed) interface condition on the pressure is used.
3. Resulting partitioned scheme is stable and accurate with no sub-time-step iterations.



Two deformable beams in an incompressible flow.

- J.W. Banks, WDH, D.W. Schwendeman, *An analysis of a new stable partitioned algorithm for FSI problems. Part II: Incompressible flow and structural shells*, J. Comput. Phys. (2014).
- Longfei Li, J.W. Banks, WDH, D.W. Schwendeman, *A stable partitioned FSI algorithm for incompressible flow and deforming beams*, J. Comput. Phys. (2016).

Incompressible flow plus light deforming beams

AMP algorithm is stable for light beams with no sub-iterations

Two beams

/Users/henshaw/movies/twoBeamsInAChannelRhos1E10.m

Incompressible flow + rigid bodies

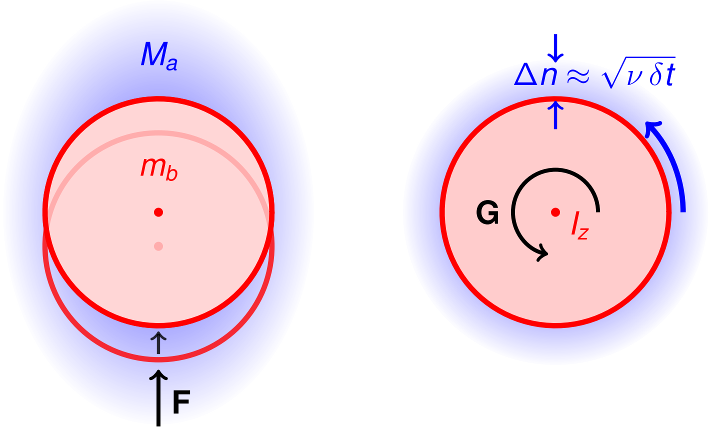
Partitioned schemes for light bodies become unstable.

- under-relaxed sub-iterations can be used to stabilize the problem
- sub-iterations are expensive, more required as bodies get lighter.

The new AMP scheme requires no sub-iterations.

- treats added-mass effects (from pressure forces on the body)
- treats added-damping effects (from viscous shear forces)

Added-mass and added-damping



Added-mass: mass of fluid moved when a force is applied to the body
Added-damping: drag on the body when a force is applied (depends on δt)

Light rigid body rising in a counter-flow

Important to address both added-mass and added-damping.

rising body

../../movies/risingDiskG8Bd0p1nu0p01.mpg

$\nu = .01$, $\rho = 1$, $\bar{\rho} = 0.1$, $R_d = 0.5$, $V_{\text{in}} = -2.4$

CHAMP: a new algorithm for conjugate heat transfer

Improved interface conditions for partitioned schemes

Conjugate heat transfer: Coupled heat transfer between fluids and solids.

Partitioned schemes: Implicit time-stepping for temperature in each domain.

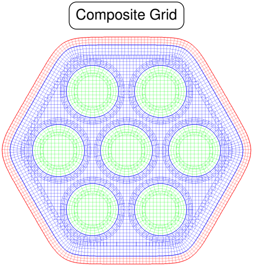
Practical problem: Depending on the material properties (thermal conductivity, thermal diffusivity) traditional interface conditions may fail or require many sub-iterations per time step.

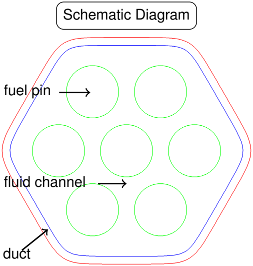
CHAMP: accurate and stable interface conditions with no sub-iterations for most practical problems.

CHAMP interface conditions combine:

- optimized-Schwartz algorithms: Robin (mixed) conditions.
- compatibility conditions derived from govering equations.

• J.W. Banks, WDH, A.K. Kapila, D.W. Schwendeman, *An Added-Mass Partitioned Algorithm for Fluid-Structure Interactions of Compressible Fluids and Nonlinear Solids*, J. Comput. Phys. (2016).





Fuel assembly

/Users/henshaw/movies/champFuelRodA

Upwind schemes for Maxwell's equations in second-order form

High-order accurate upwind schemes are stable on overset grids

Maxwell's equations as a second-order wave equation (SOWE)

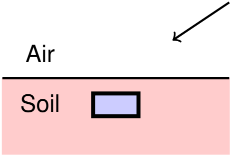
$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \Delta \mathbf{E}$$

- traditional schemes for the SOWE require artificial dissipation
- choosing the coefficient of dissipation is tricky on overset grids
- thin boundary layer grids require more dissipation

- new upwind schemes automatically add dissipation
- schemes can be any order of accuracy
- analysis confirms stability on overset grids
- numerical results confirm the accuracy and stability

Scattering from a buried rectangular body

Electromagnetic scattered field E_x from a chirped incident plane-wave pulse



Embedded body

/Users/henshaw/movies/ebAnnulusG8E