An added mass partitioned algorithm for rigid bodies and incompressible flows

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Overset Grid Symposium
Mukilteo, WA
October 19, 2016

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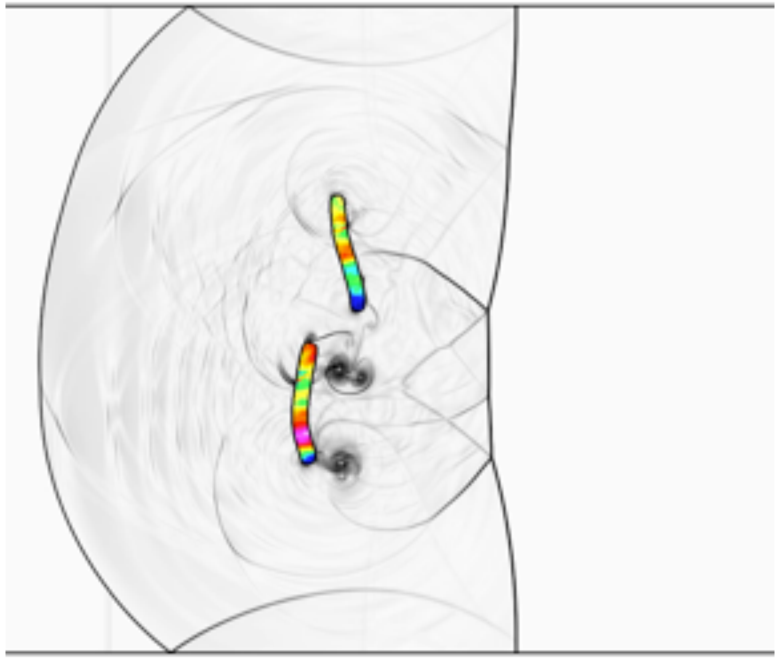
In recent work we have focused on large deformation and/or displacement FSI using partitioned solvers



- Component solvers remain independent
  - can use existing solvers
  - no need to solve (or precondition) a coupled implicit system
- Can naturally take advantage of disparate time scales
  - e.g. mixing implicit and explicit integration
- High levels of algorithmic concurrency
  - maps well to modern and emerging computers

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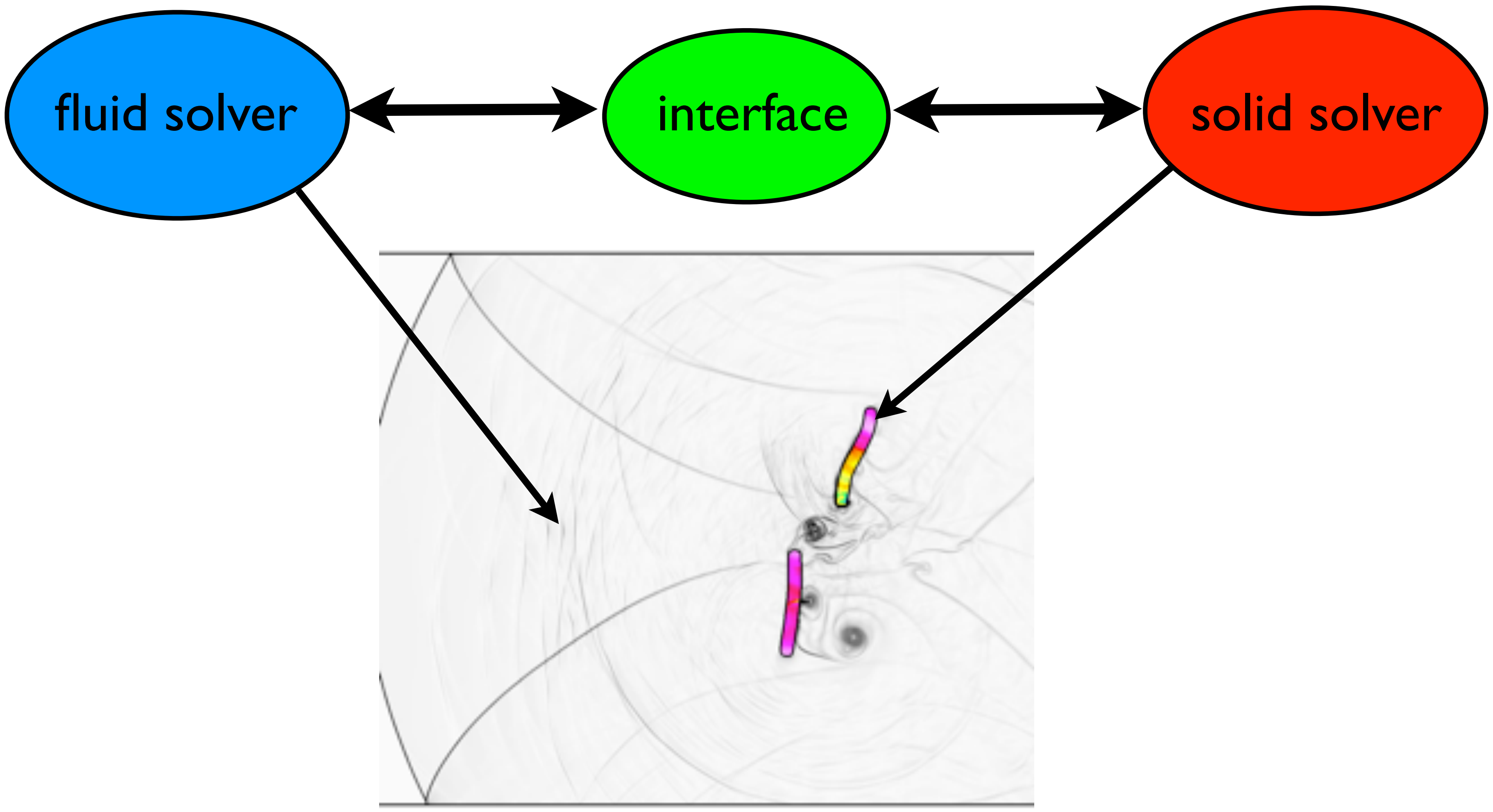

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Traditional partitioned schemes have suffered from "added-mass instabilities" for solids which are sufficiently light when compared to the fluid

- Traditional partitioned FSI algorithms (Cirak, et. al. 2007, Bungartz and Schafer 2006)
  1. advance fluid (using interface velocity/displacement from the solid)
  2. advance solid (apply fluid forces to the solid)
  3. possibly iterate with under-relaxation to convergence

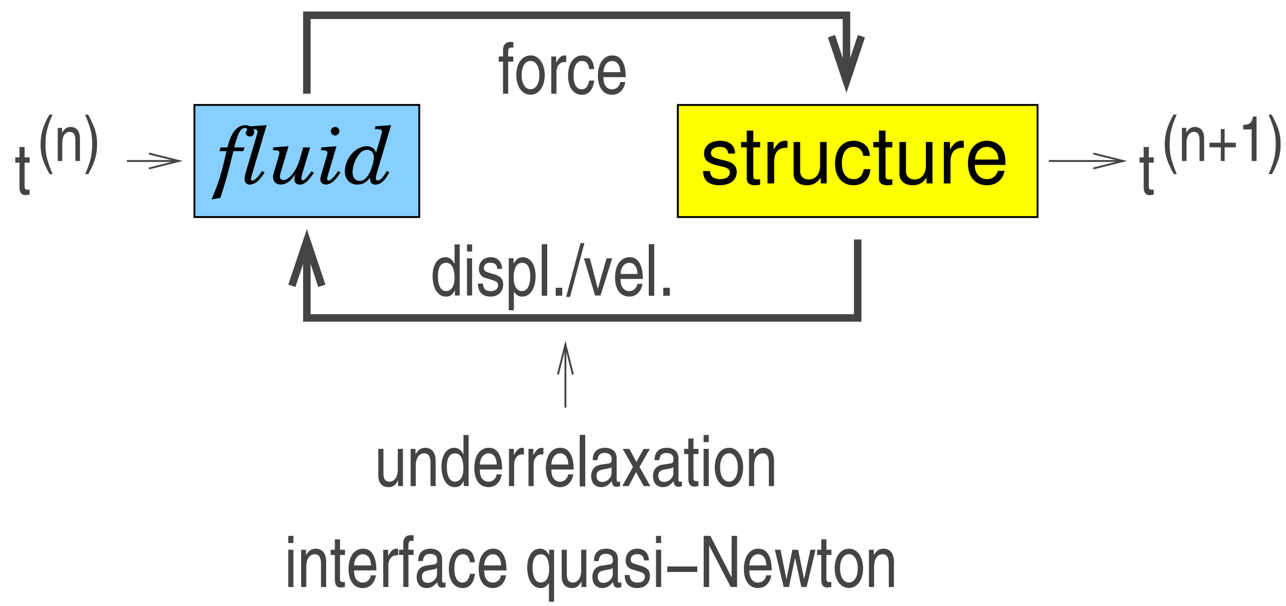
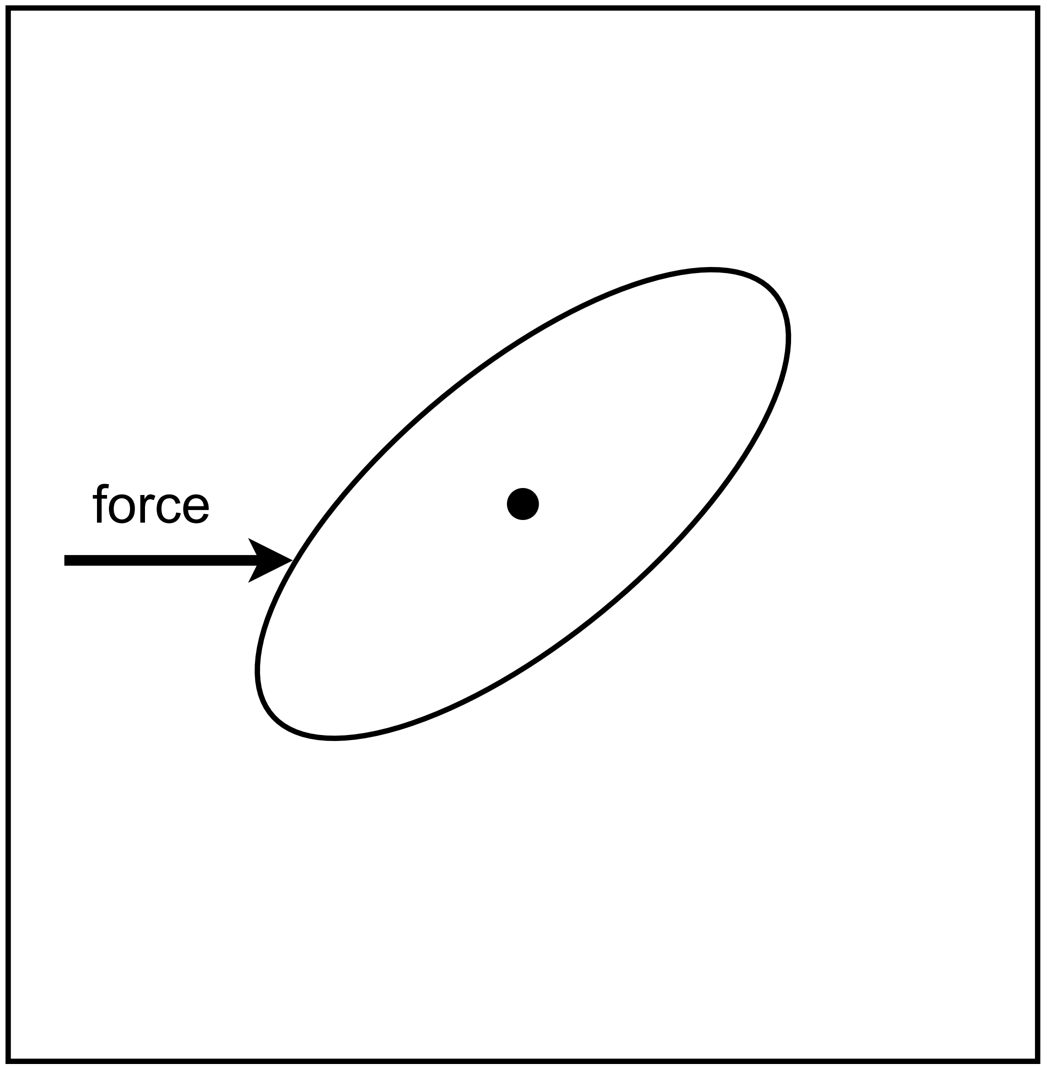


diagram from Keyes et. al. 2012

- Some analysis of added-mass instabilities can be found in the literature, for example
  - Causin, Grebeau, and Nobile, 2005 (stability with relaxation)
  - Gretarsson, Kwatra, and Fedkiw 2011 (semi-monolithic formulations)

The origin of added-mass instabilities is that the effect of displaced fluid is not appropriately accounted for in the numerical algorithms

in a vacuum



Body simply moves according to Newton's laws of motion

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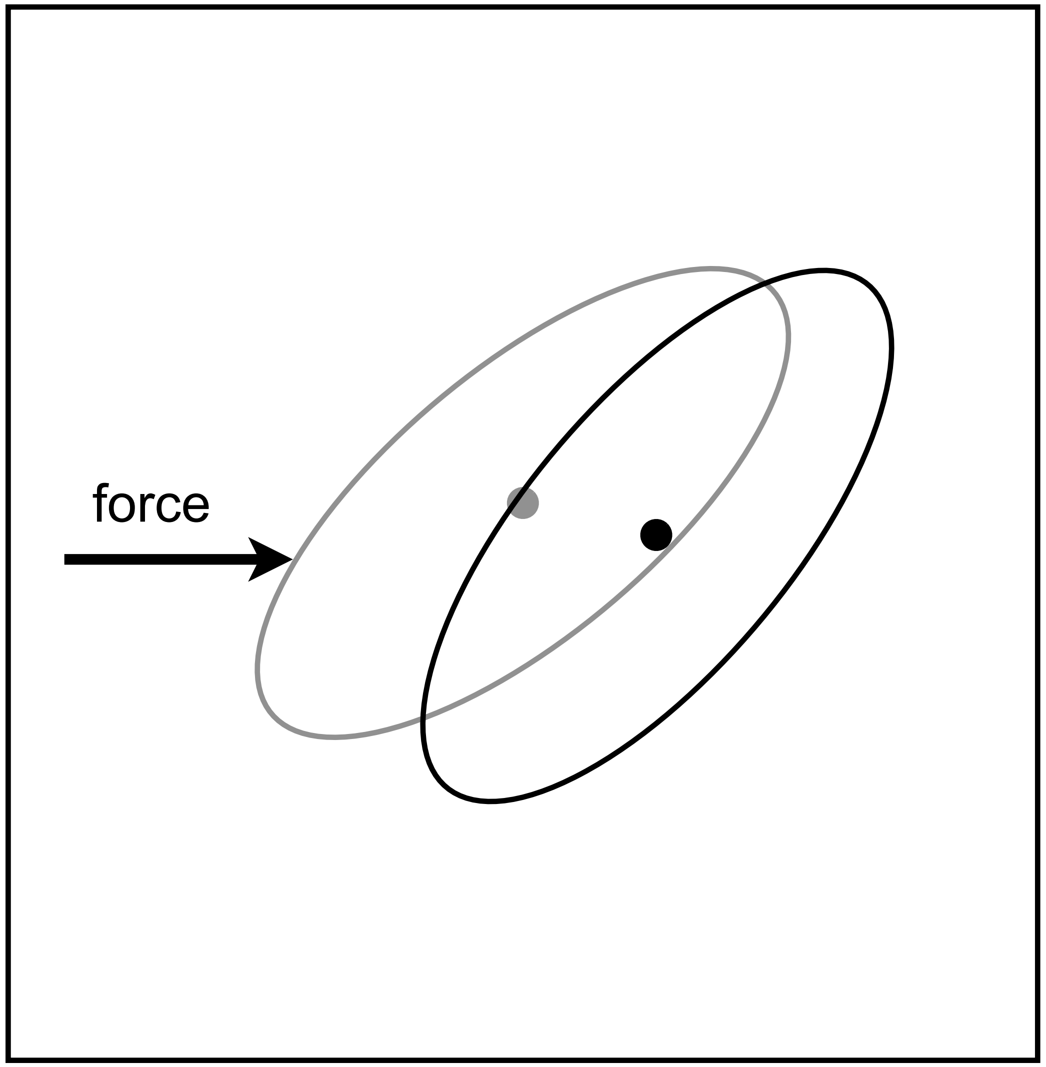

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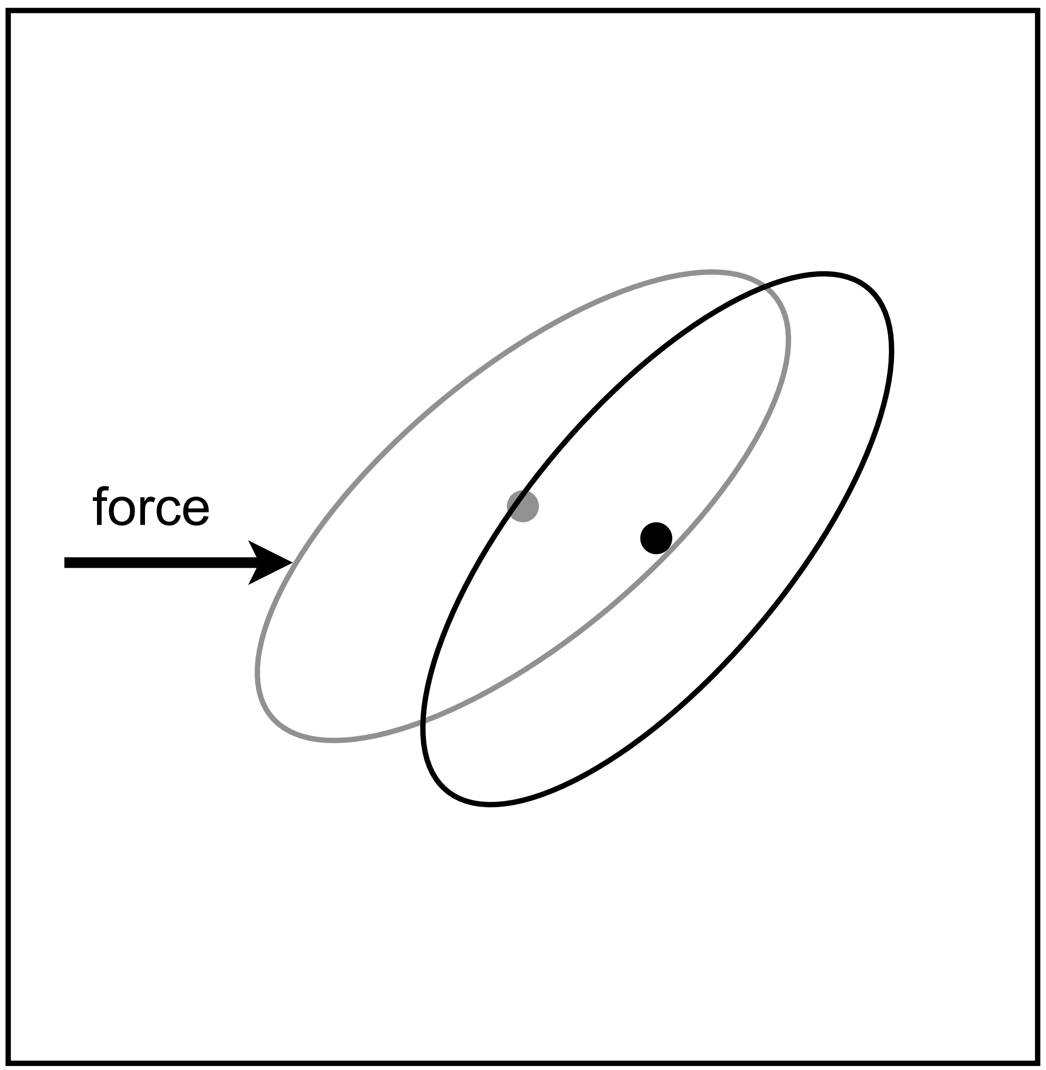

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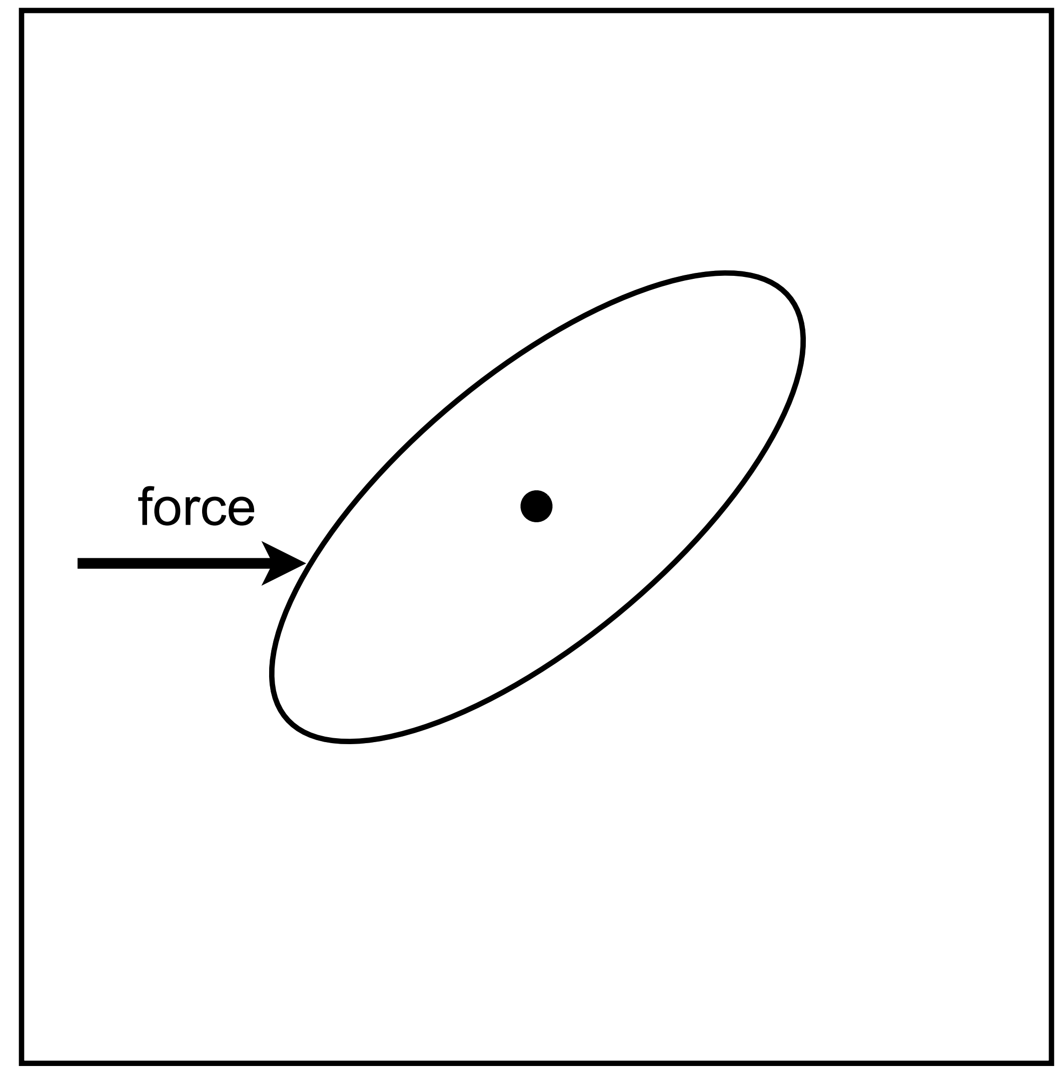

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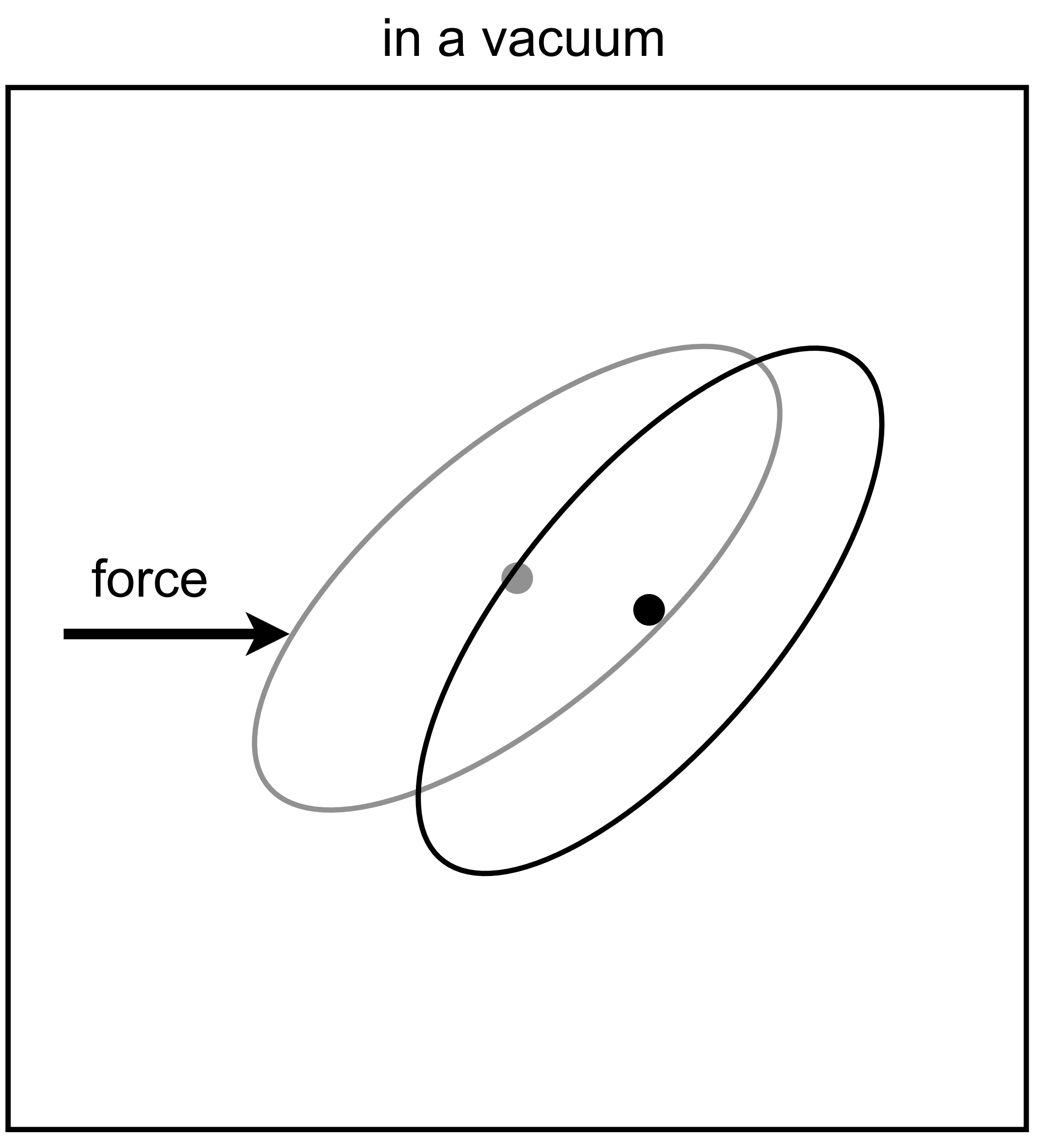

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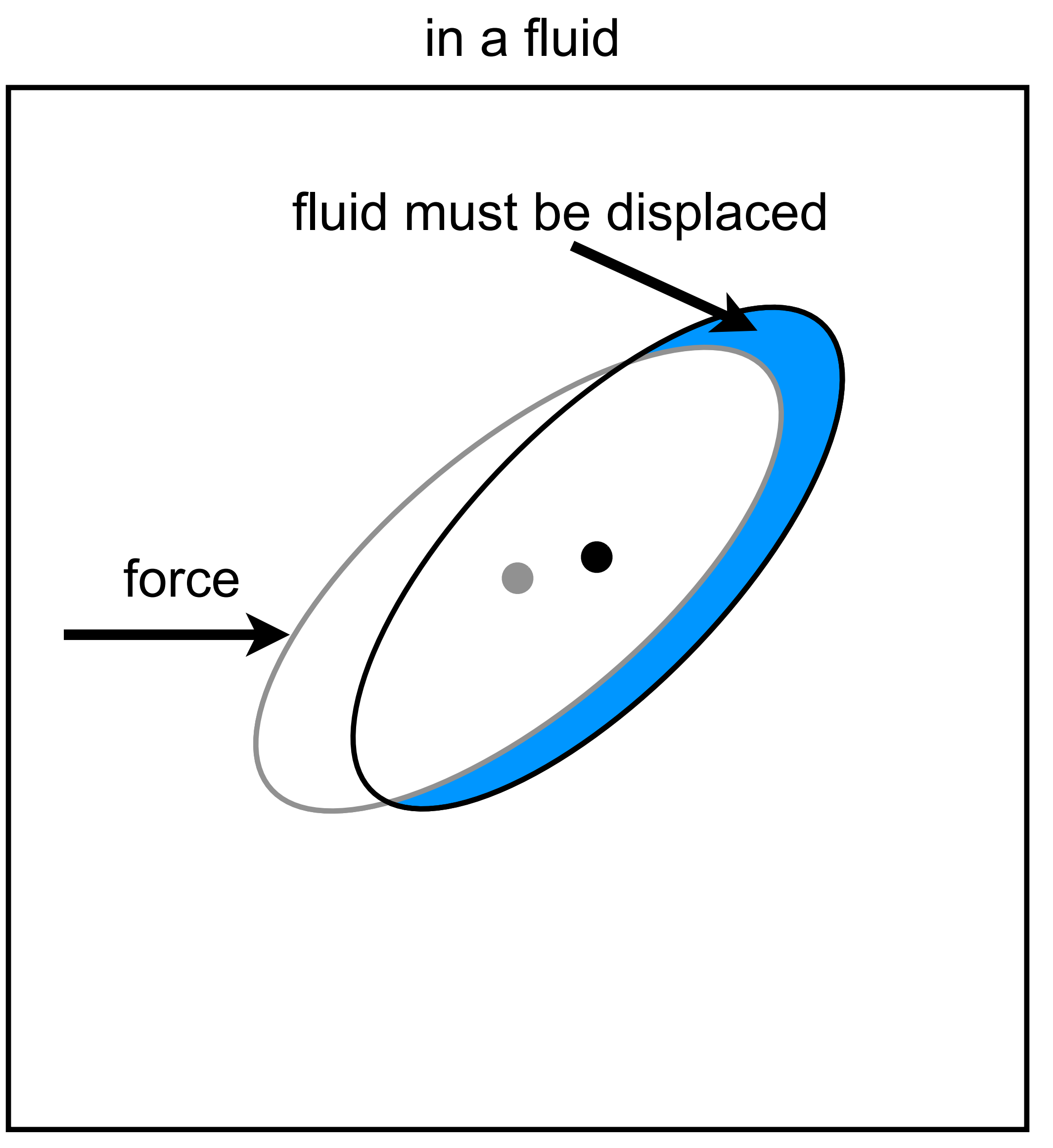

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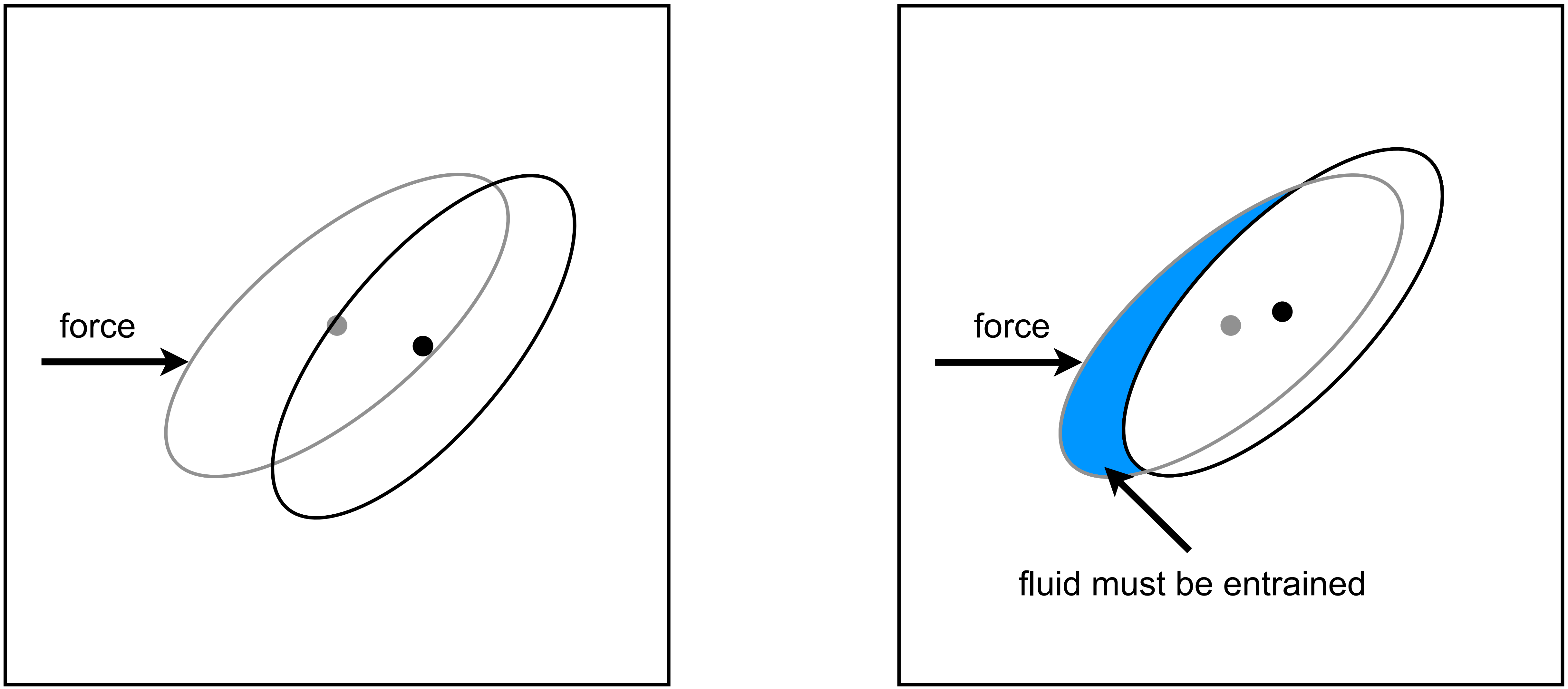

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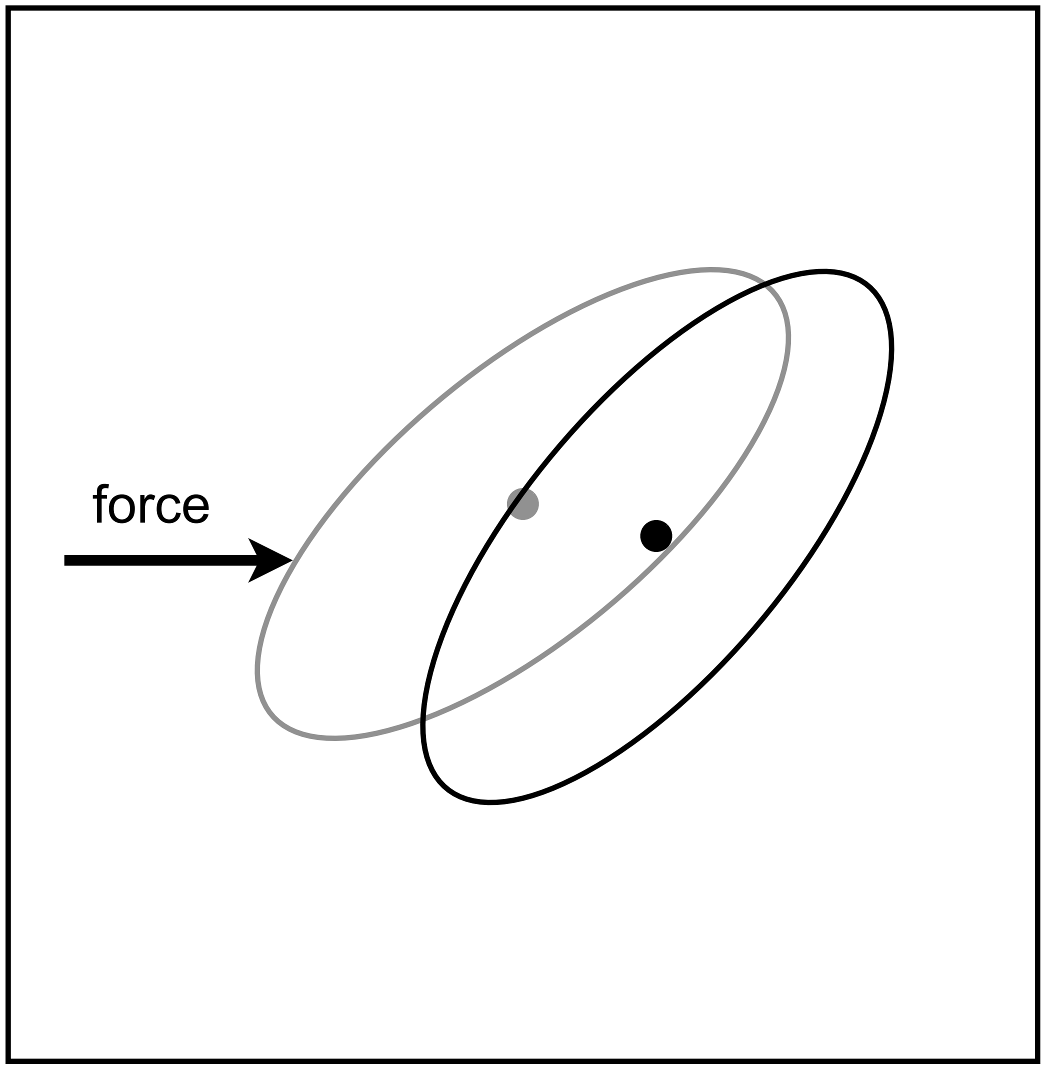

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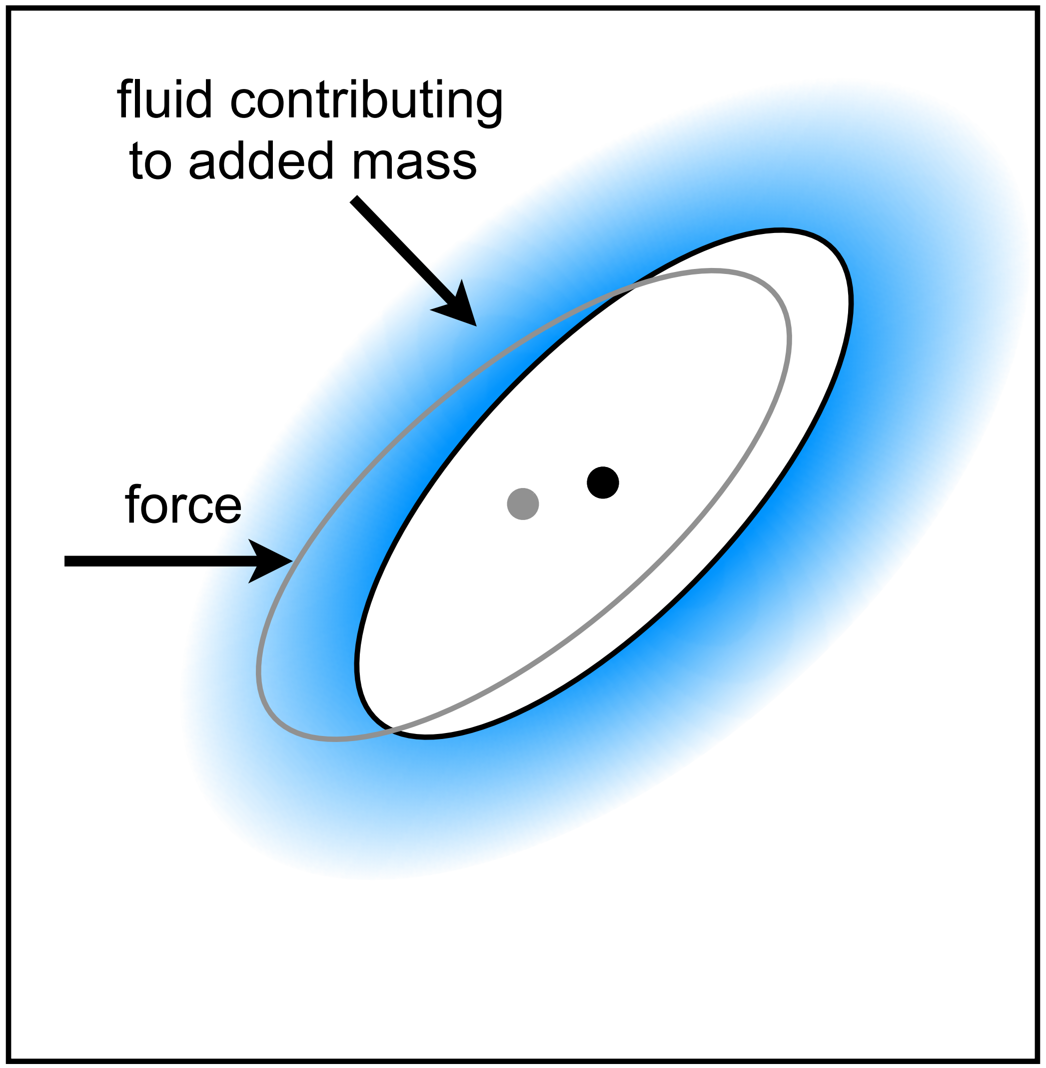
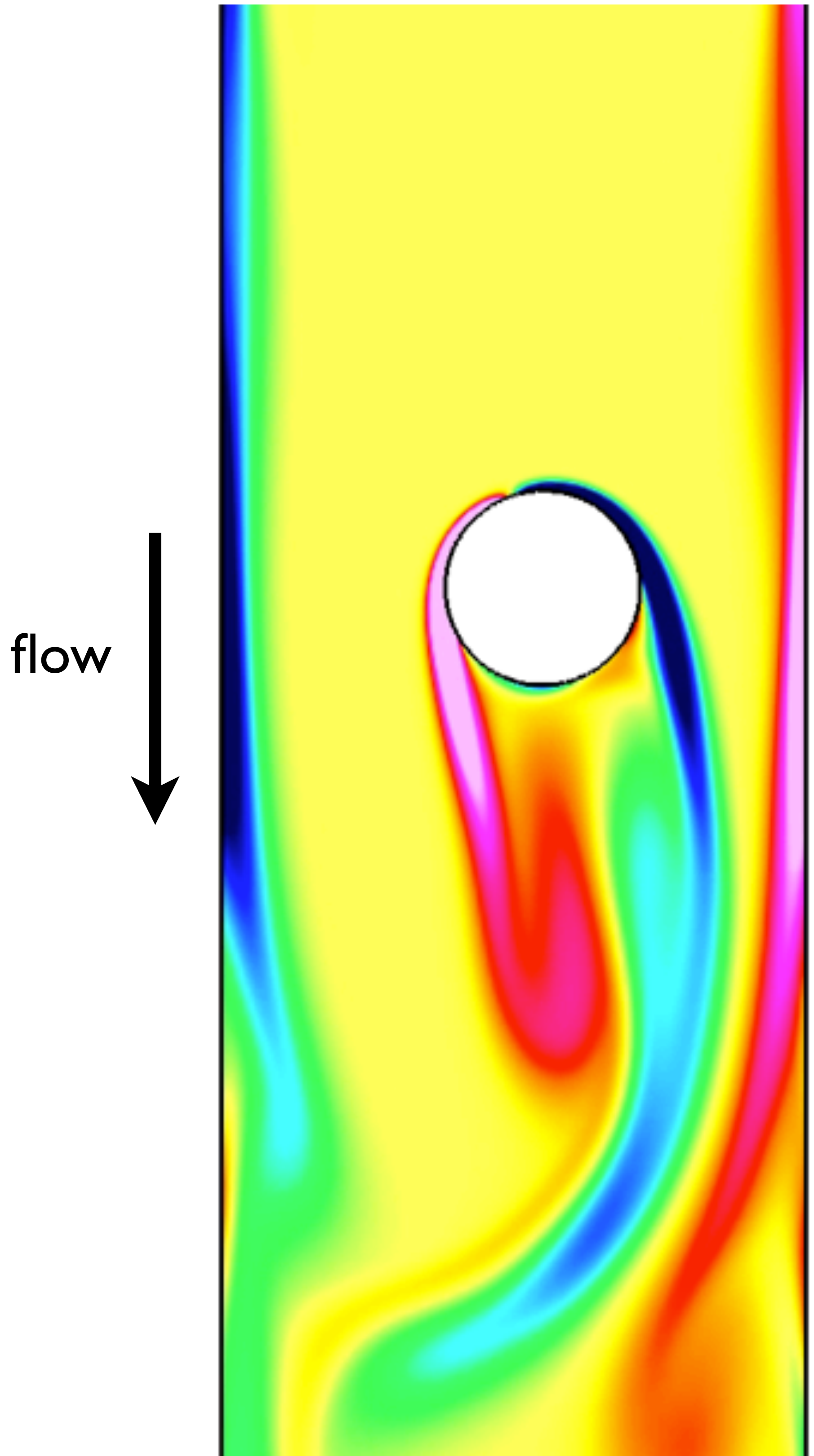


Body must displace and entrain fluid to move and therefore appears more massive than in vacuum ... the so called "added mass"

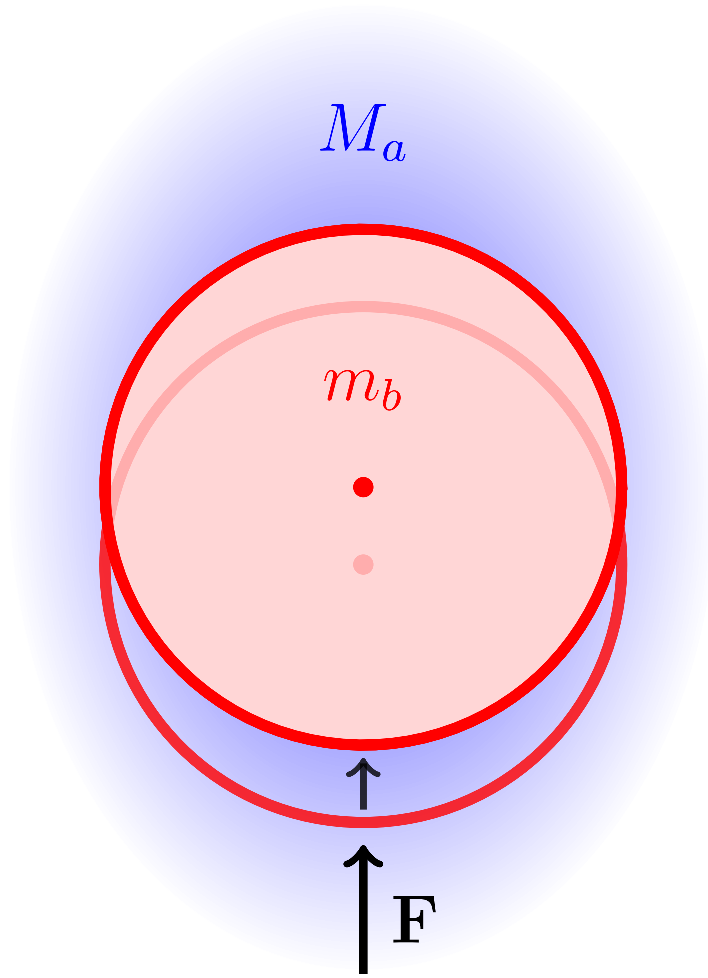
As a concrete motivating example consider a rising rigid body in counterflow

- Incompressible Navier-Stokes
- Light rigid body

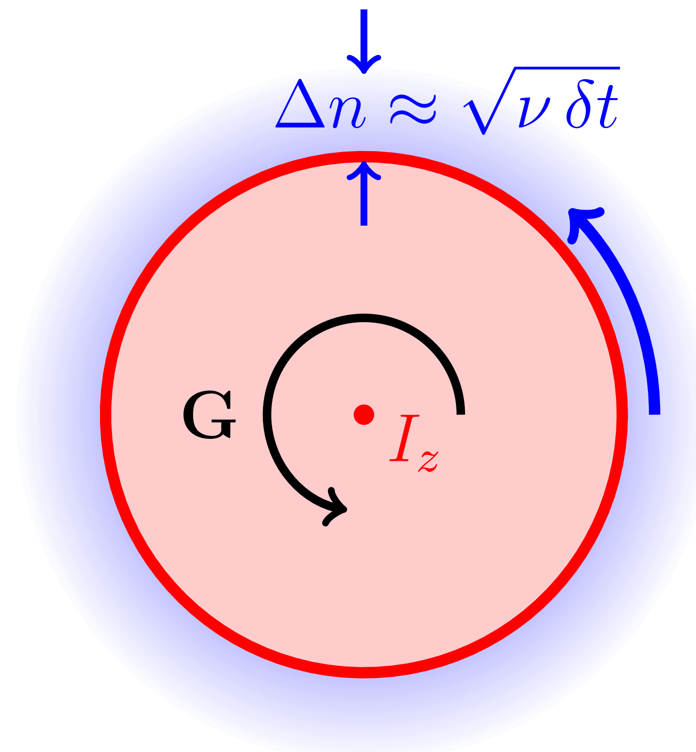


This case has both strong added-mass and added-damping effects

Added Mass

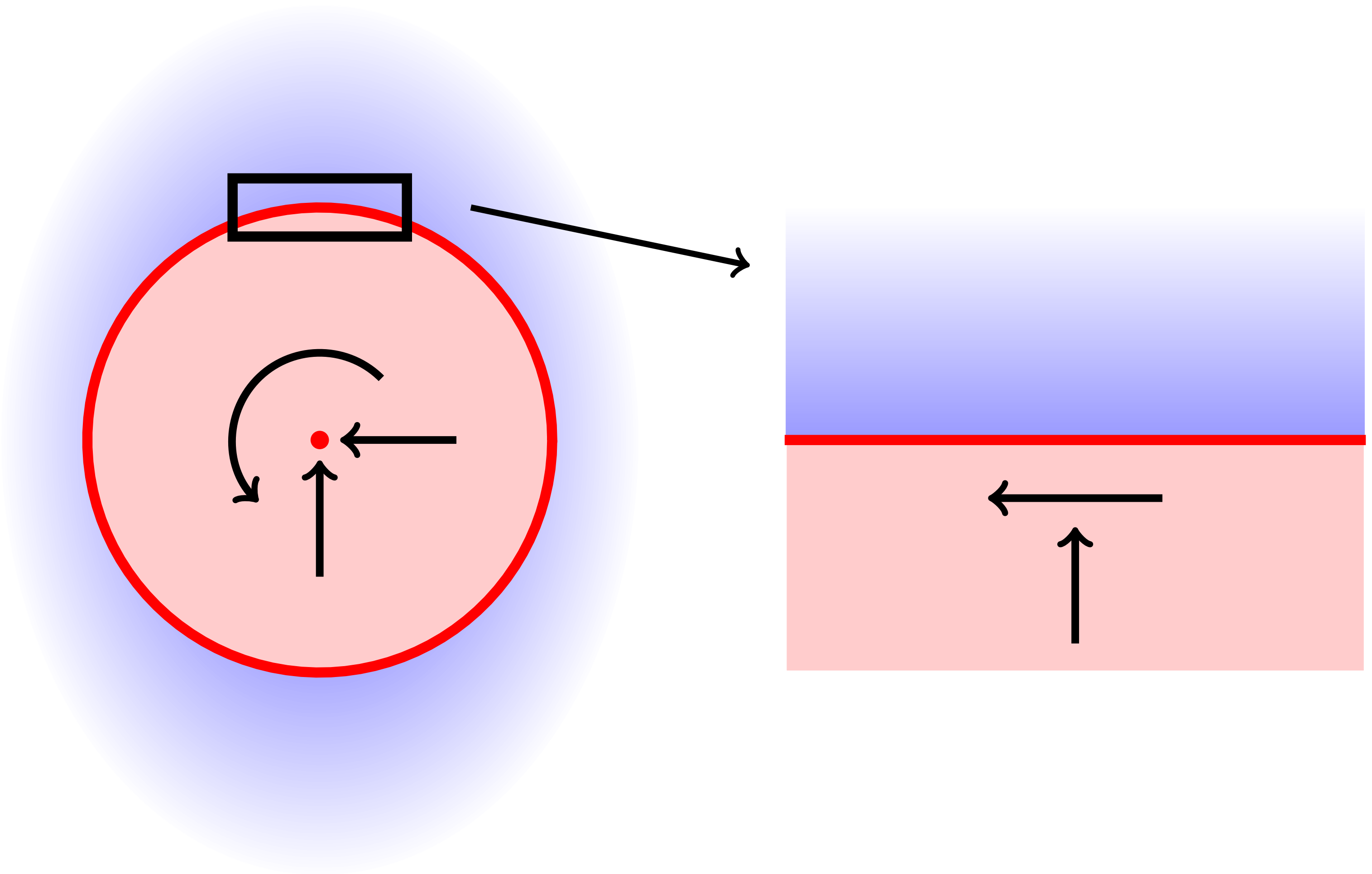


Added Damping



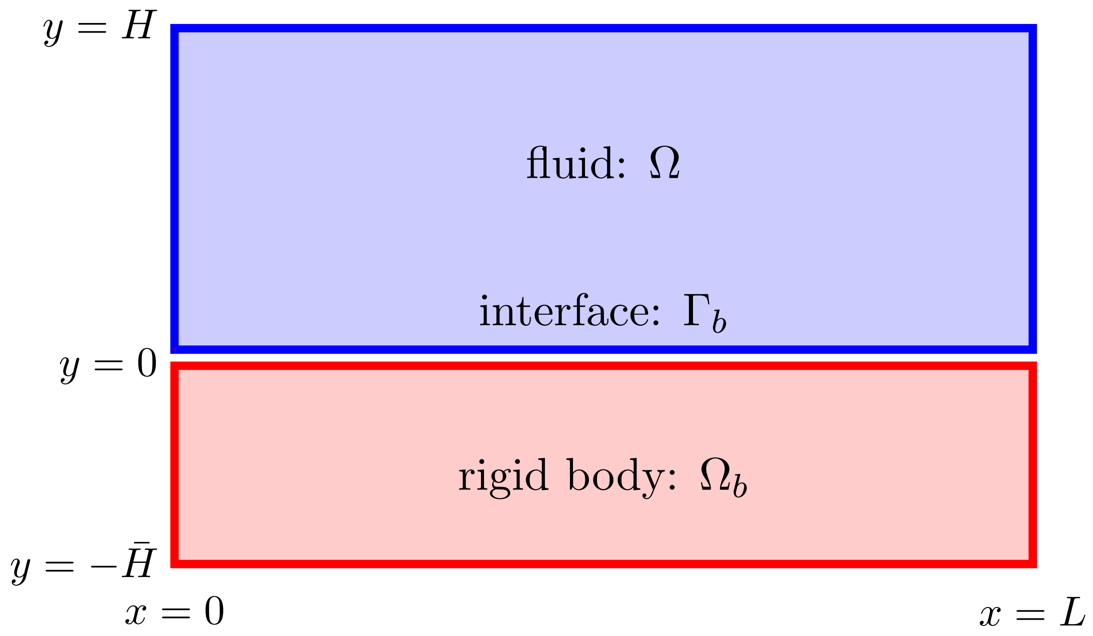
- Added mass relates to increased apparent mass owing to fluid displacement
  - i.e. the local geometry occupied by solid changes
- Added damping relates to increased apparent inertia owing to viscous fluid drag
  - i.e. the local geometry remains fixed

To understand these effects in isolation we derive extremely simple models by localizing and linearizing the problem near the interface



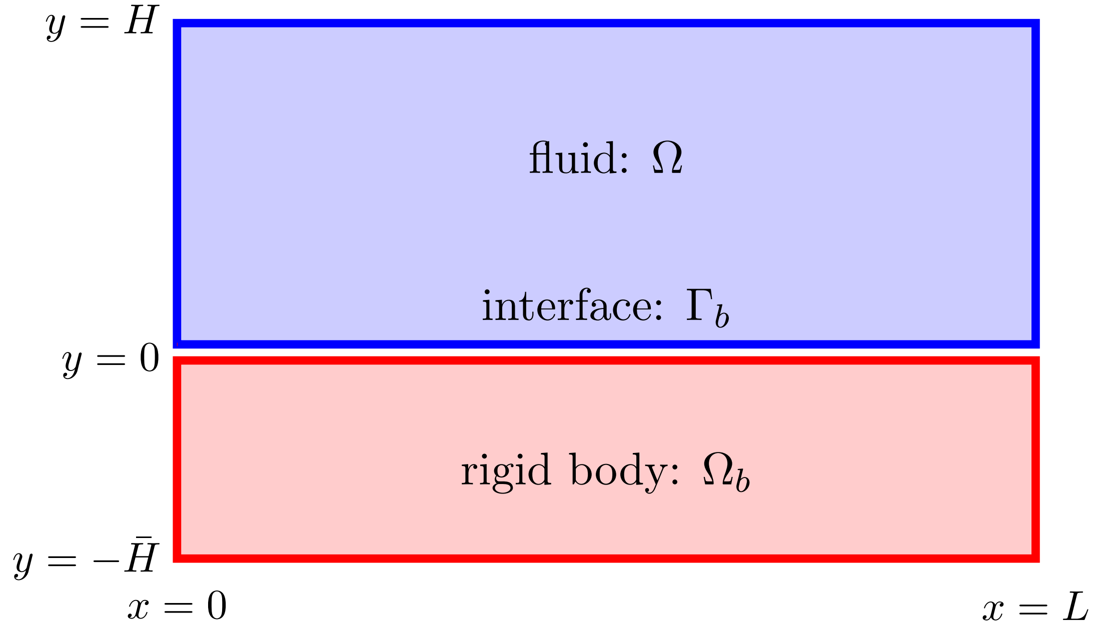
- y-translations relate to added-mass effects
- x-translations relate to added-damping effects

The resulting model is used to motivate our new AMP algorithms and discuss the performance of traditional partitioned (TP) schemes



$$
\begin{aligned}
\text{Fluid:} \quad & \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = \mu \Delta \mathbf{v}, && \mathbf{x} \in \Omega, \\
& \nabla \cdot \mathbf{v} = 0, && \mathbf{x} \in \Omega, \\
\text{Rigid body:} \quad & m_b \, a_u = \int_0^L \mu \frac{\partial u}{\partial y}(x,0,t)\, dx + g_u(t), && \\
& m_b \, a_v = -\int_0^L p(x,0,t)\, dx + g_v(t), && \\
\text{Interface:} \quad & \mathbf{v}(x,0,t) = \mathbf{v}_b(t), && x \in [0, L],
\end{aligned}
$$

An added-mass model problem is derived by considering vertical motions



$$\begin{cases} \rho \dfrac{\partial v}{\partial t} + \dfrac{\partial p}{\partial y} = 0, & y \in (0, H), \\ \dfrac{\partial v}{\partial y} = 0, & y \in (0, H), \\ m_b \dfrac{d v_b}{dt} = -\displaystyle\int_0^L p \, dx, & \\ v(0,t) = v_b, \quad p(H,t) = p_H(t), & \end{cases}$$

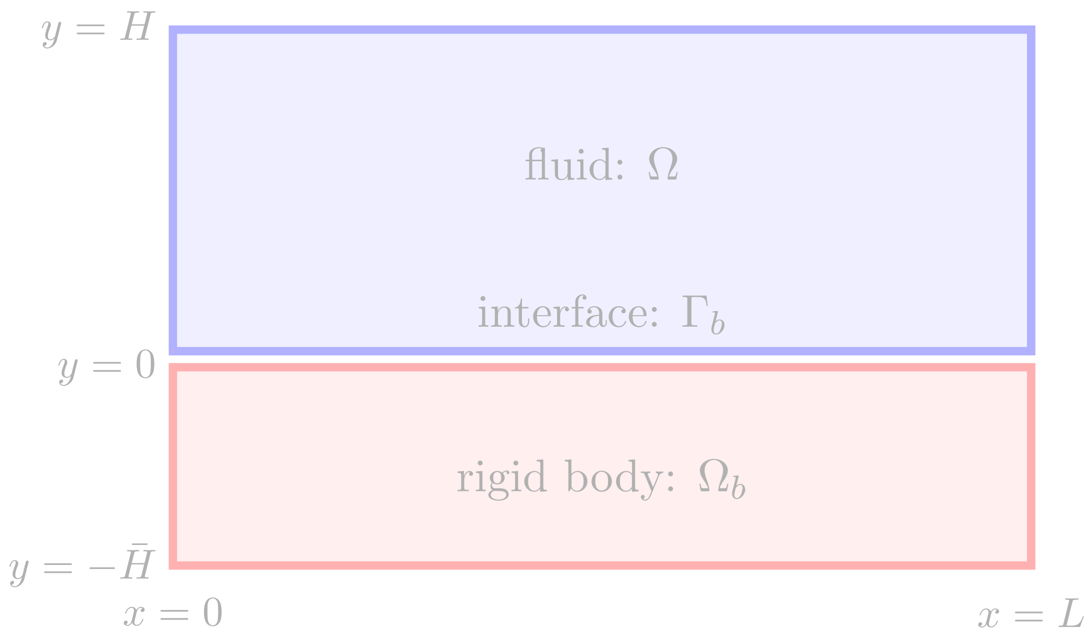
- The AMP scheme matches the vertical accelerations at the interface

$$\left. \frac{\partial v}{\partial t} \right|_{y=0} = \frac{d v_b}{dt} = a$$

- And applies a generalized Robin condition to the fluid pressure equation

$$\rho a + \frac{\partial p}{\partial y} = 0 \qquad\qquad m_b a = \int_0^L p \, dx$$

The resulting AMP scheme is stable for any finite mass, while the traditional scheme suffers



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$$

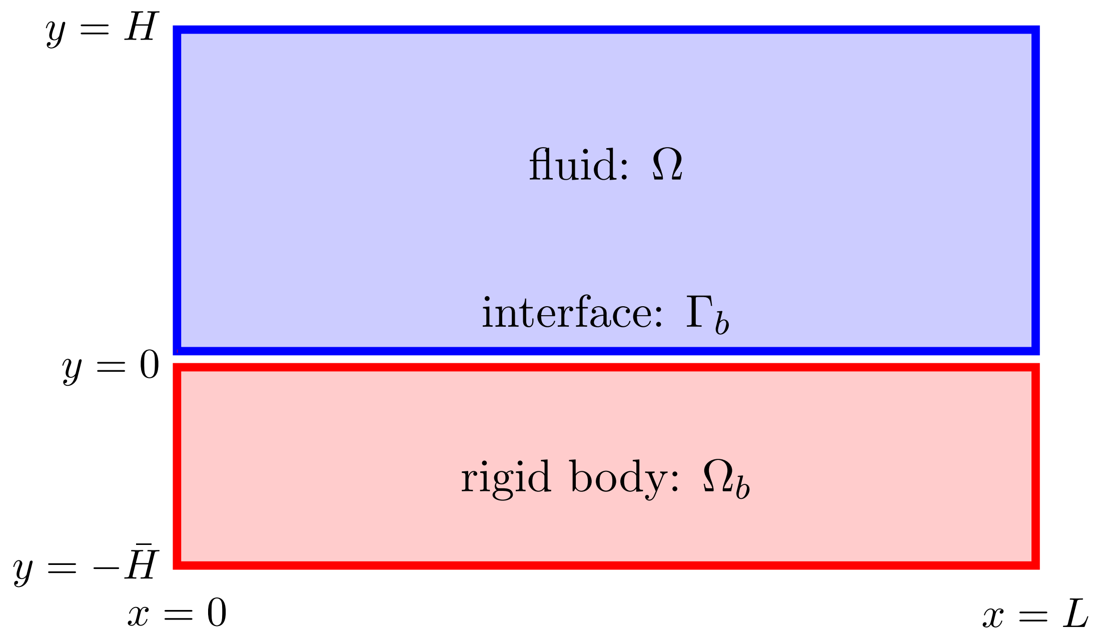
- The added mass for this problem is easily identified as

$$M_a = \rho L H$$

- Thm: The 2nd order accurate AMP scheme for the added-mass model problem is stable provided $m_b + M_a$ is bounded away from zero.

- Thm: The 2nd order traditional partitioned scheme is stable if and only if $m_b > M_a$

An added-damping model problem is derived by considering horizontal motions



$$
\begin{cases}
\rho \dfrac{\partial u}{\partial t} = \mu \dfrac{\partial^2 u}{\partial y^2}, & y \in (0,H), \\
m_b \dfrac{du_b}{dt} = \mu \displaystyle\int_0^L \dfrac{\partial u}{\partial y}(0,t)\,dx, & \\
u(0,t) = u_b(t), \quad u(H,t) = u_H(t), &
\end{cases}
$$

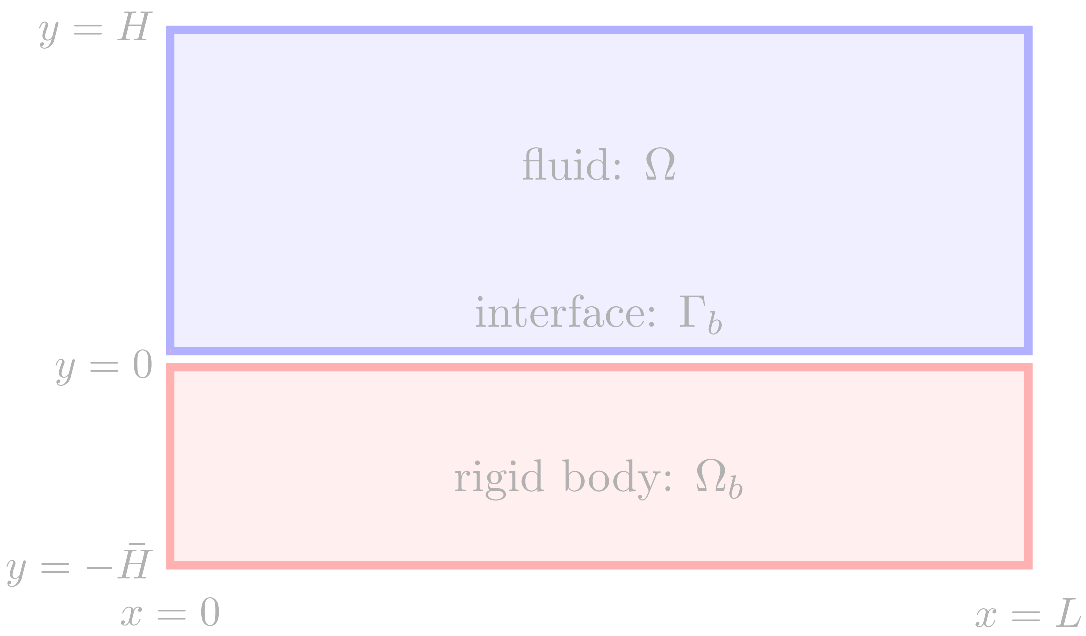
- The AMP scheme uses the exact solution to form the discrete approximation

$$\mu \int_0^L \frac{\partial u}{\partial y}(0,t)\,dx \approx -\beta \mathcal{D} u_b$$

$$\mathcal{D} \approx \mu \int_0^L \frac{1-e^{-\delta}}{\Delta y} \qquad \delta = \frac{\Delta y}{\sqrt{\nu \Delta t/2}}$$

here δ is a ratio of viscous length scales, and $\mathcal{D}$ is an added-damping coefficient (in 3D these become added-damping tensors)

The AMP scheme with extra velocity projection is stable even for massless bodies



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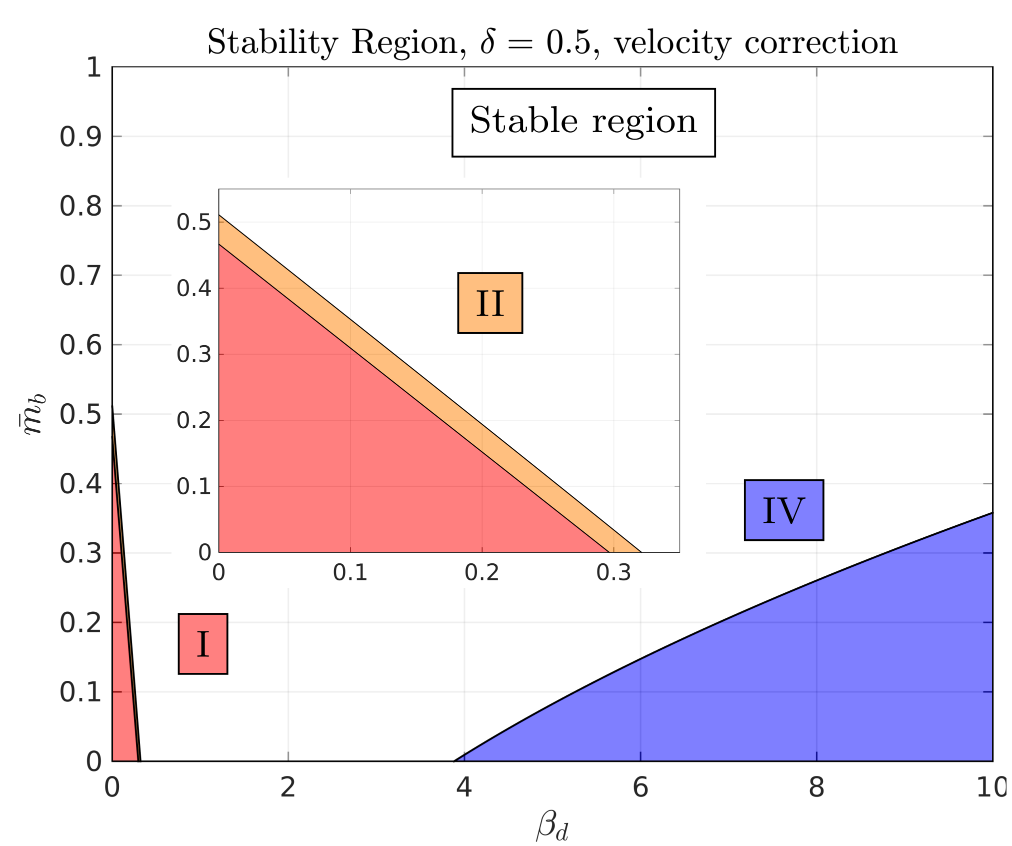
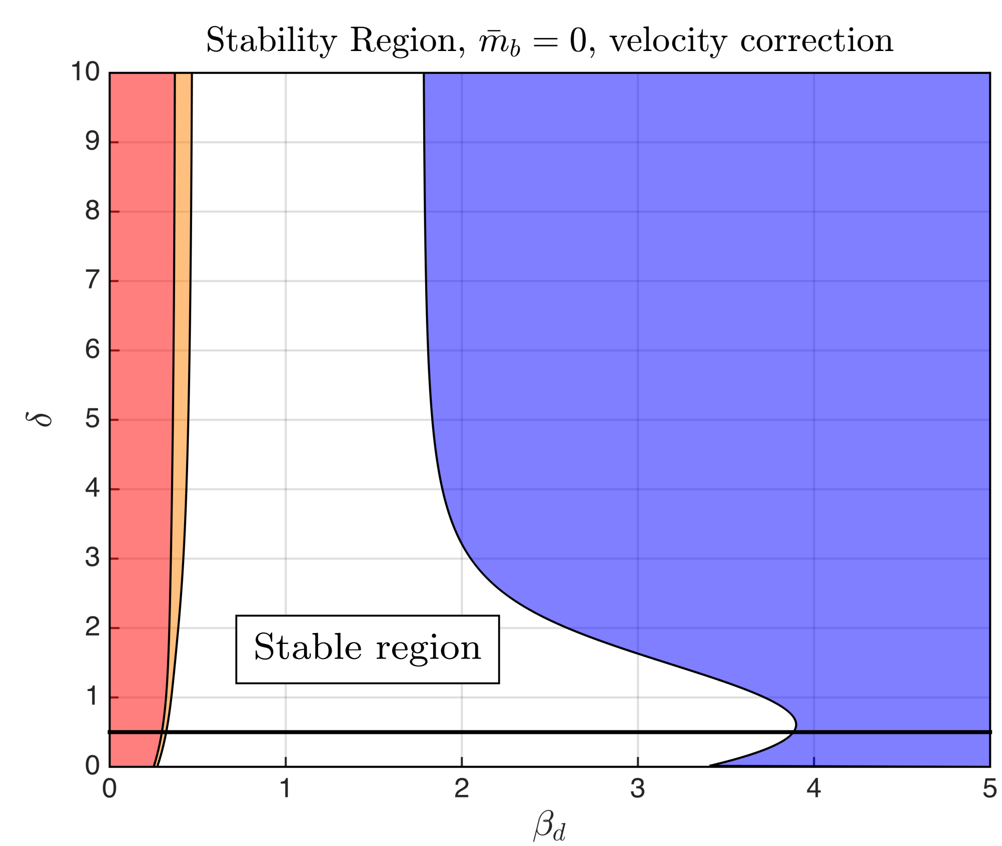
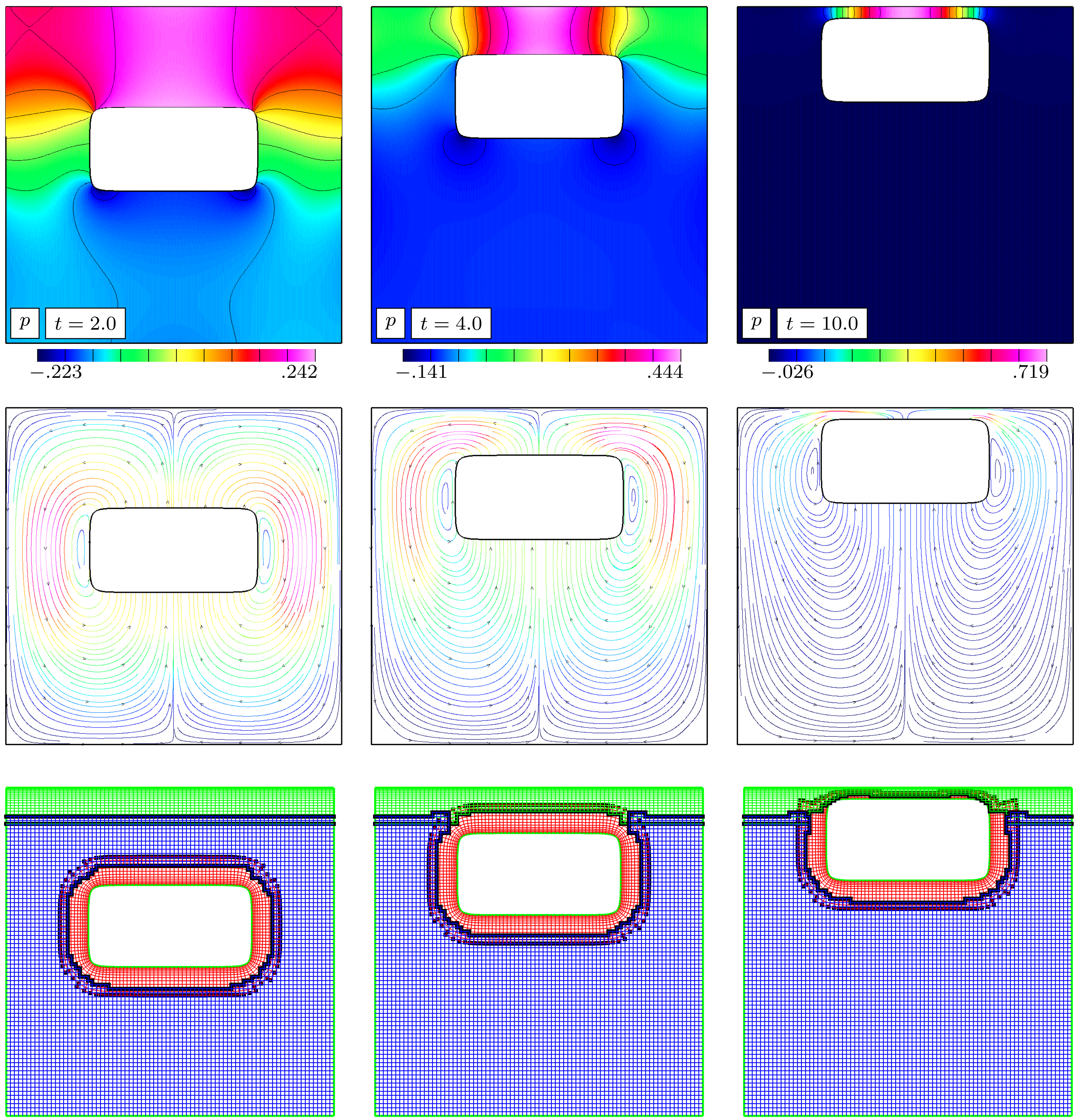




The AMP-RB scheme is implemented in Overture and is found to be stable against both added-mass and added damping instabilities without iteration

$$\rho_b = .001$$



By comparison, the traditional partitioned scheme requires ~85 under-relaxed iterations to provide comparable results

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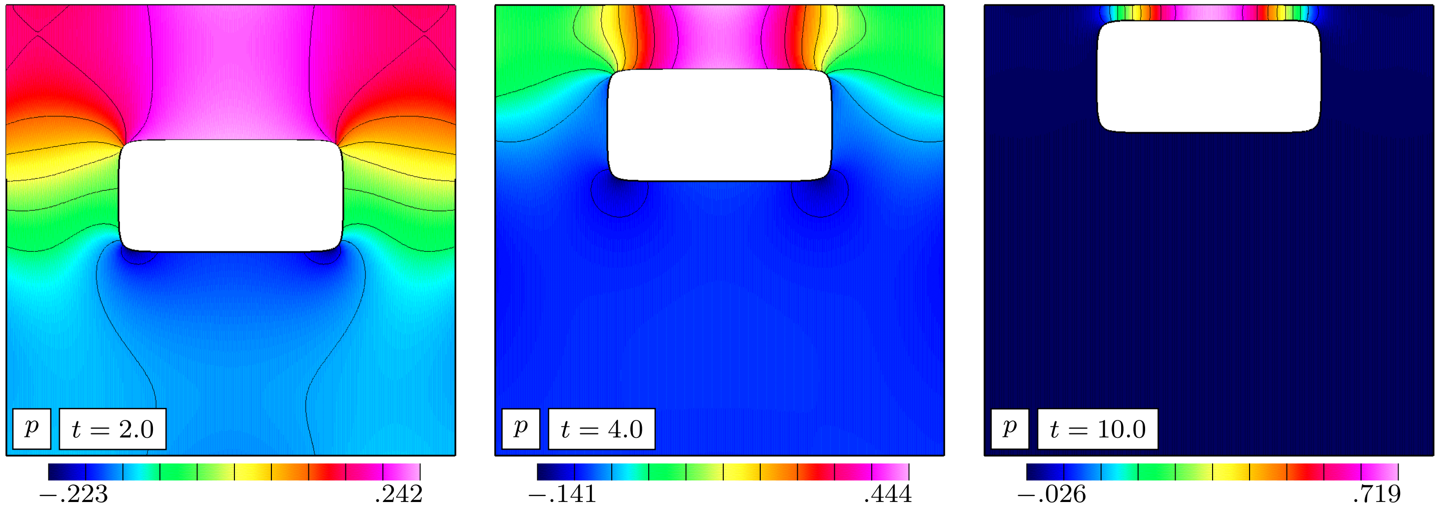
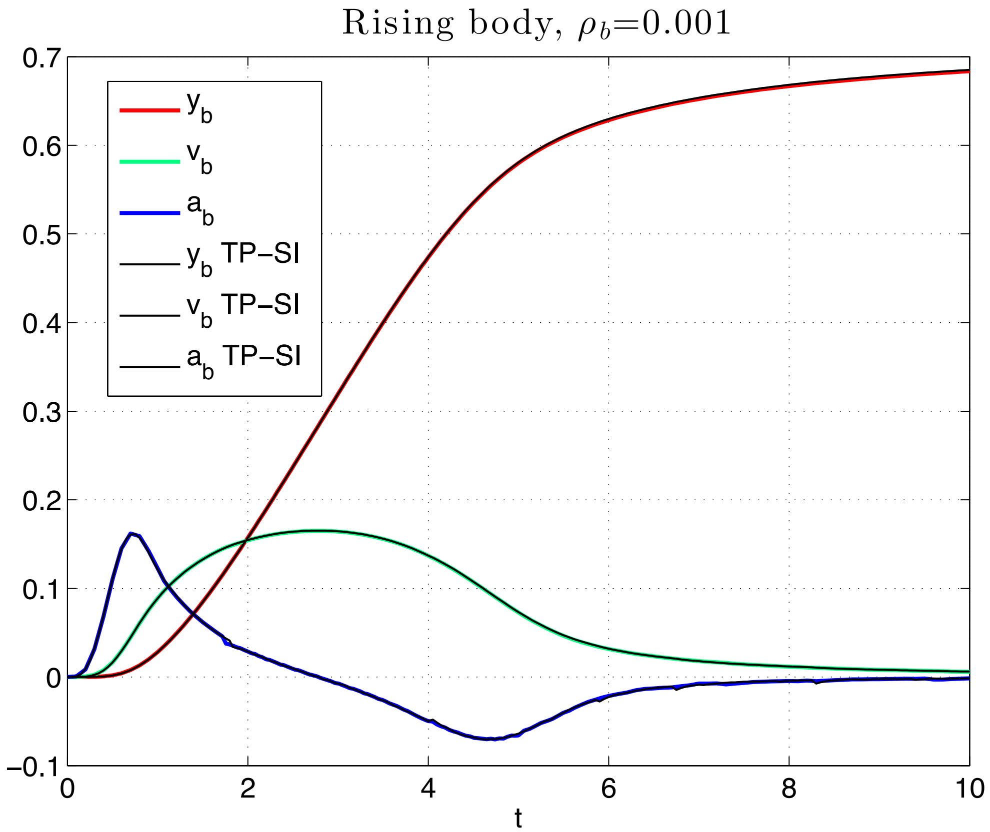
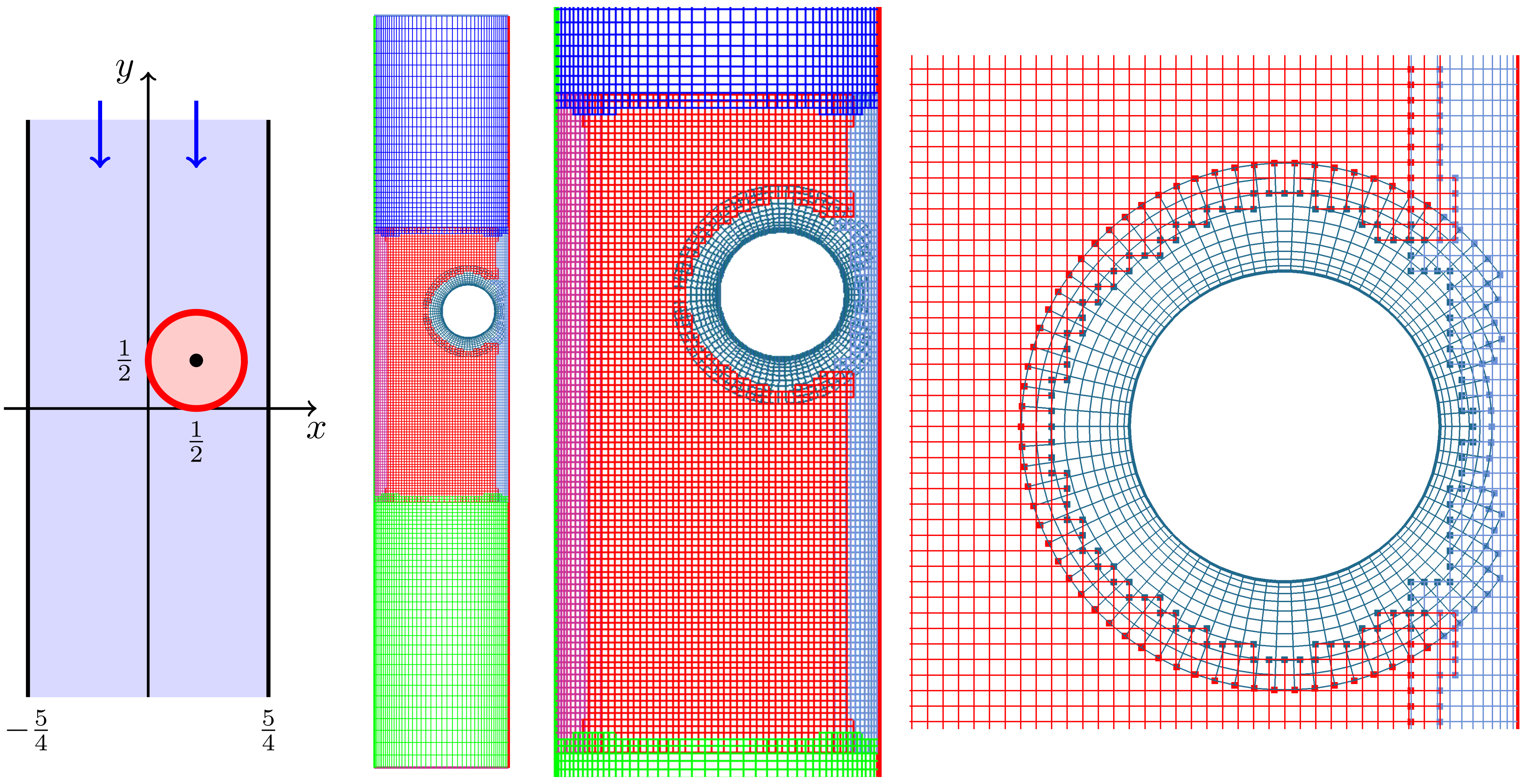




A more challenging case is that of a light cylinder rising in counterflow

- The AMP-RB scheme is again stable without any iteration
- Traditional partitioned scheme needs ~2000 sub iterations to stabilize

$$\rho_b = .001$$



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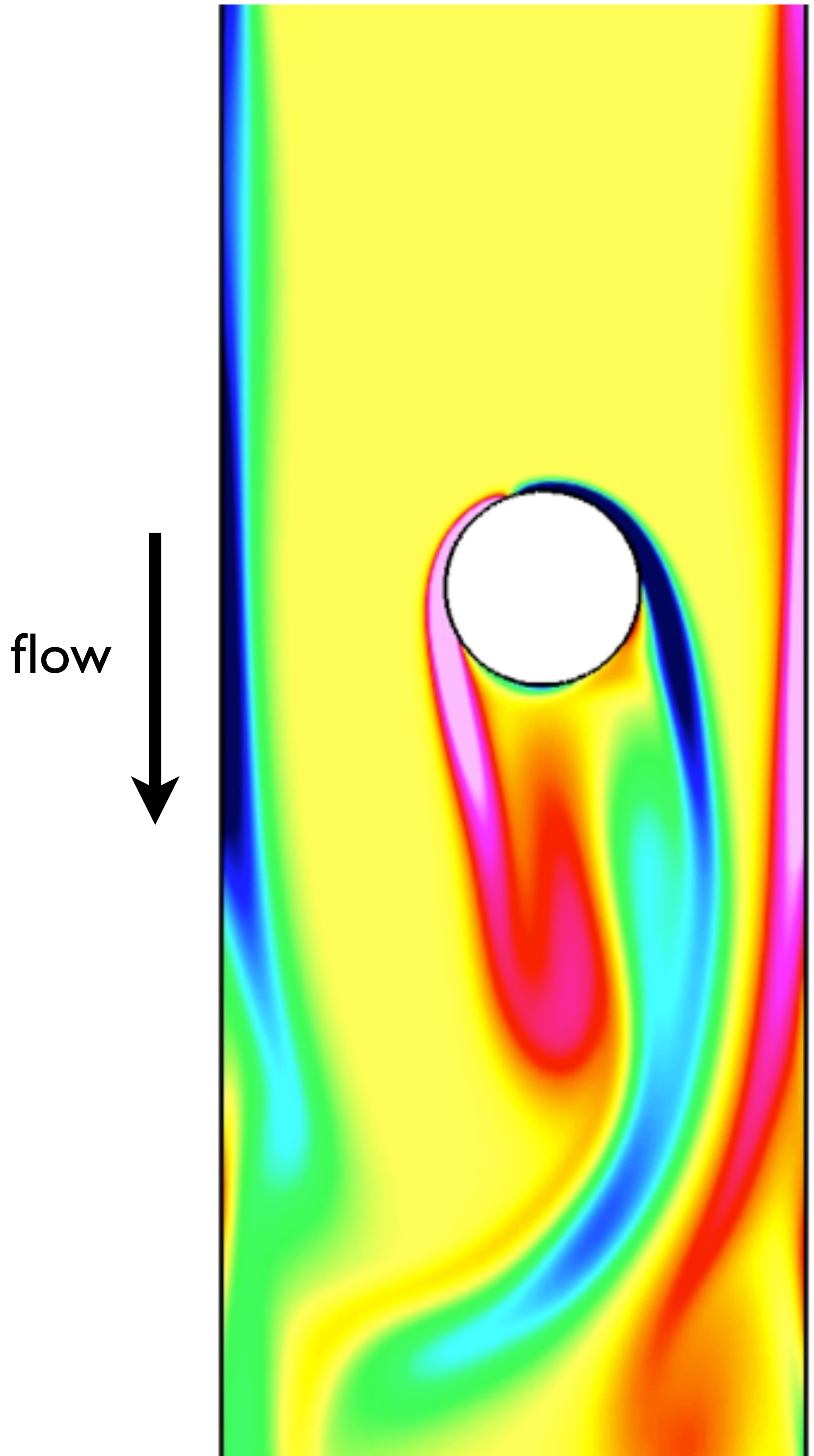
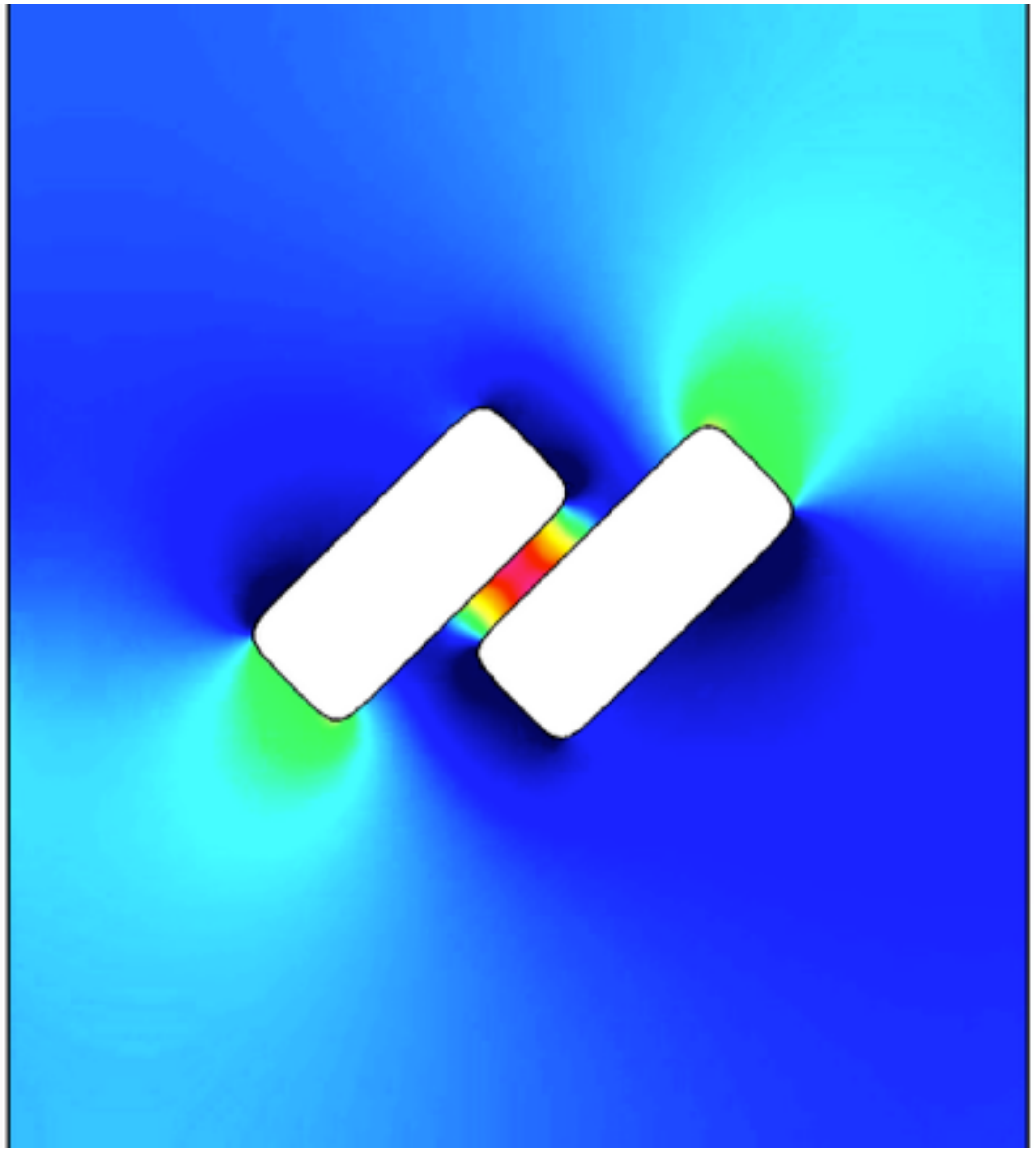


Extensions to multiple bodies presents no particular challenges, and the scheme is robust even in the presence of both light and heavy bodies



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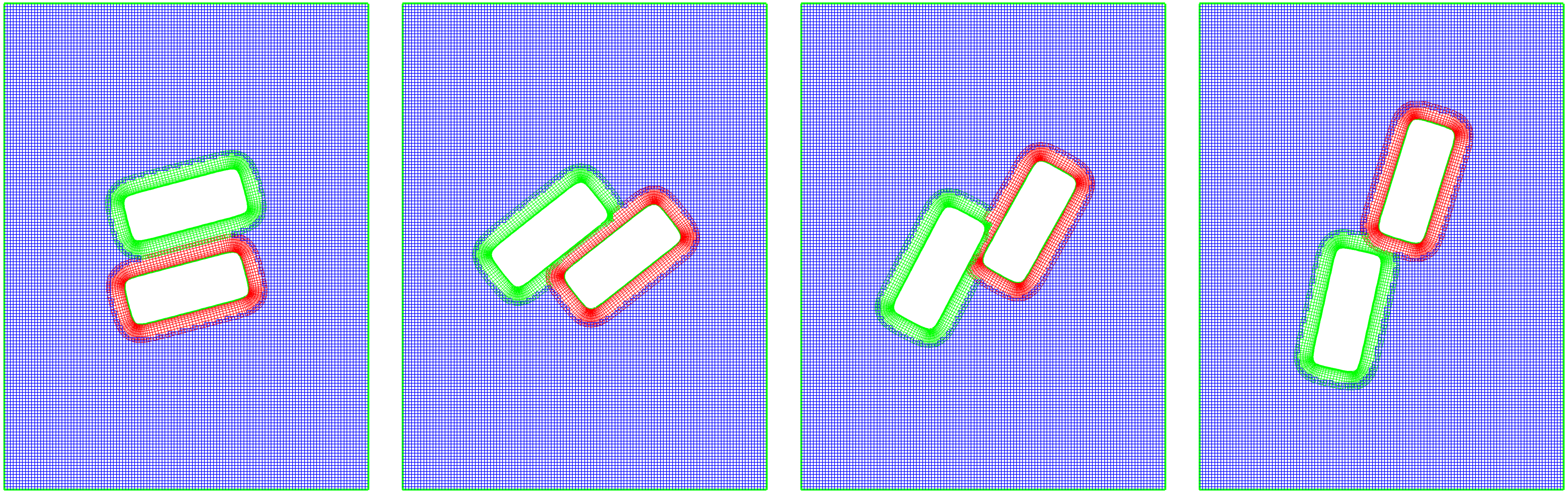

Summary

- Careful analysis of simplified model problems motivates our stable and 2nd order accurate AMP-RB scheme for incompressible flows

- Stability analysis in simple geometries shows excellent stability properties even in the un-iterated form

- Implementation within Overture illustrates the utility of the approach for both light and heavy bodies

Future Work

- implement in 3D

- investigate the alternate added-mass potential formulation

- New FSI regimes (incompressible/incompressible, compressible/beams)